Use of Precession Modulation for Nutation Control Spin-Stabilized Spacecraft

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Nomenclature

a_0	=	accelerometer gain
a	=	accelerometer output
I_x, I_y, I_z	=	moments of inertia about principal axes
N_0	=	torque
T_s	=	spacecraft spin period
α	=	$-1/\tau+i\Omega$, complex energy dissipation
λ	=	angular position of control thruster
$\Delta\Phi$	=	$\Omega\Delta t$, thrust pulse duration, in radians
Δt	=	thrust pulse duration, in seconds
μ	=	accelerometer angular position
Φ_0	=	nutation phase angle at initiation of thrust
σ	=	measure of moment of inertia ratio
τ	=	energy dissipation time constant
θ	=	nutation angle
$\omega(t)$	=	transverse angular velocity
ω_0	=	initial condition for ω in Eqs. (6) and (7)
ω_s	=	spacecraft spin rate
Ω	=	nutation frequency

Theme

THIS paper derives relations for nutation induced in a spinning spacecraft by periodic precession thrust pulses. By using the idea that nutation need only be observed just before each precession thrust pulse, a tedious continuous-time derivation reduces to a simple discrete-time derivation which can be easily solved through the use of z-transforms. The derived results are used to develop two precession control laws which use the precession maneuver to control nutation.

Contents

In 1968 Grasshoff[1] published the original work on a control law which senses and removes nutation. Grasshoff's equation, (Eq. 2 of Ref. 1, with $N_0e^{i\lambda}$ replacing N) for the transverse angular velocity of the spinning spacecraft

$$\dot{\omega}(t)+i\Omega\omega(t)=N_0e^{i\lambda} \tag{1}$$

with a control thrust for (t_2-t_1) seconds is given by

$$\omega(t_2)=\omega(t_1)e^{-i\Omega(t_2-t_1)}+(N_0e^{i\lambda}/i\Omega)[1-e^{-i\Omega(t_2-t_1)}] \tag{2}$$

For small pertubations, the increase in nutation angle, θ, is generally represented[3] as

$$\theta(t)=\theta_0e^{t/\tau} \tag{3}$$

in which τ is the time constant of energy dissipation. Due to a linear relationship between θ and $\omega(t)$, energy dissipation can be included in Eq. (1) to give

$$\dot{\omega}(t)+[-(1/\tau)+i\Omega]\ \omega(t)=N_0e^{i\lambda} \tag{4}$$

Nutation can be sensed by an accelerometer and can be removed by properly timing a small thruster[1]. Precession can be effected by using the same thruster used for nutation control. For a precession maneuver of equally spaced thrust pulses Δt wide with period $T_s=2\pi/\omega_s$, Eq. (1) becomes

$$\dot{\omega}(t)+i\Omega\omega(t)=\begin{cases} N_0e^{i\lambda}, nT_s\leq t<n\,T_s+\Delta t \\ 0, nT_s+\Delta t\leq t<(n+1)T_s \end{cases} \quad n=0,1,2,... \tag{5}$$

The continuous solution for Eq. (5) is complicated by the pulse train. The inverse Laplace transform of the Laplace transform of Eq. (5) is difficult because the pole at $s=-i\Omega$ cannot be separated by a contour from the poles at $\pm i2n\pi/T_s$, $n=1,2,3,...$ (see Ref. 4).

The solution of Eq. (5) is straightforward if it is sufficient to know only the result at the discrete points, nT_s, just prior to the next thrust pulse. Thus, Eq. (5) can be changed to a first-order difference equation. (See Ref. 5). By taking the z-transform and then the inverse z-transfrom, the discrete-time solution for Eq. (5) is

$$\omega[nT_s]=\omega_0e^{-i\Omega(nT_s)}+\frac{N_0e^{i\lambda}}{i\Omega}(1-e^{i\Omega\Delta t})e^{-i\Omega T_s} \times\left[\frac{1}{1-e^{-i\Omega T_s}}+\frac{e^{-i\Omega(nT_s)}}{1-e^{i\Omega T_s}}\right] \tag{6}$$

The discrete-time solution for Eq. (4) with the forcing function of Eq. (5) can be obtained similarly as

$$\omega[nT_s]=\omega_0e^{-\alpha(nT_s)}+\frac{N_0e^{i\lambda}}{\alpha}(1-e^{-\alpha\Delta t})e^{-\alpha T_s} \times\left[\frac{1}{1-e^{-\alpha T_s}}+\frac{e^{-\alpha(nT_s)}}{1-e^{\alpha T_s}}\right] \tag{7}$$

where $\alpha=-(1/\tau)+i\Omega$. The conclusion from Eq. (7) is that with energy dissipation, nutation due to unmodulated precession is unbounded.

Because precession and nutation control can use the same thruster, precession pulses can be selected that concurrently reduce nutation. The range of Φ_0, which is the sector of the nutation cycle over which nutation will be reduced, can be derived from Eq. (2). After trigonometric manipulation, replacement of $\omega(t_1)$ by $\omega(t_1)e^{-i\Phi_0}$ and $\Omega(t_2-t_1)$ by $\Delta\Phi$

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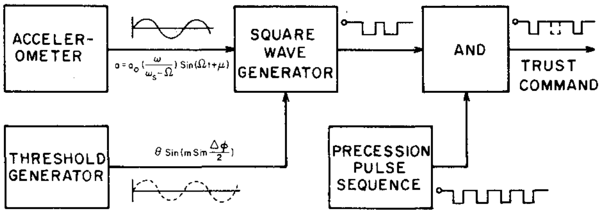

Fig. 1 Modulated precession nutation control.

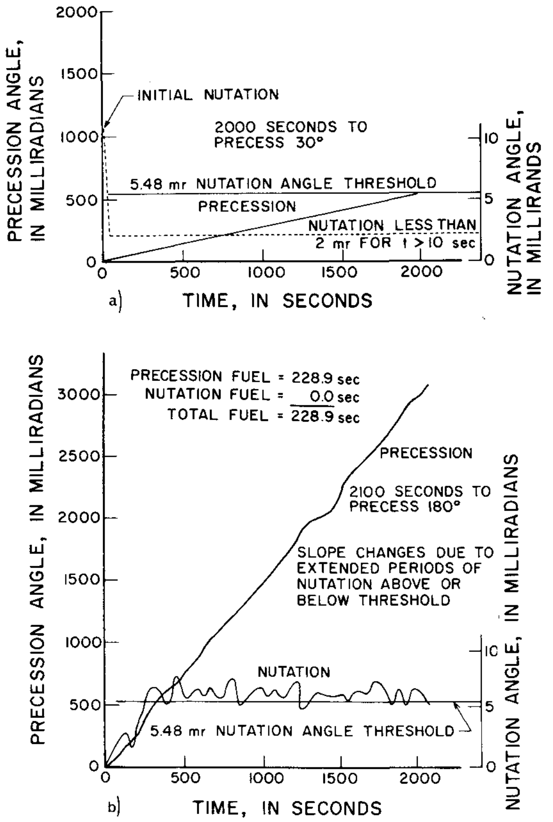

Fig. 2 a) SMS simulation for full modulated precession nutation control; b) SMS simulation for full-precession-to-threshold modulated precession nutation control.

gives

$$\omega(t_2)=\omega(t_1)e^{-i\Phi_0}e^{-i\Delta\Phi}+\frac{2N_0}{\Omega}\sin\left(\frac{\Delta\Phi}{2}\right)e^{i[\lambda-(\Delta\Phi/2)]} \qquad (8)$$

Equation (8) is Grasshoff's, but restructured so that the range of Φ_0 for which $\omega(t_2)\leq\omega(t_1)$ can be determined. Thus,

$$\arccos\,[-m\sin(\Delta\Phi/2)]_{\mathrm{II}}-\lambda-(\Delta\Phi/2)$$
$$\leq\Phi_0\leq\arccos\,[-m\sin(\Delta\Phi/2)]_{\mathrm{III}}-\lambda-(\Delta\Phi/2) \qquad (9)$$

where $\arccos\,[\cdot]_{\mathrm{II}}$ and $\arccos\,[\cdot]_{\mathrm{III}}$ refer to angles in the II and III quadrants and $m=(N_0/\Omega)[1/\omega(t_1)]$

To summarize, Eq. (9) can be used to modulate, or gate, the precession pulse train so that only precession pulses are allowed which reduce nutation. In fact, a nutation sensing accelerometer and a threshold derived from Eq. (9) can be used.

Figure 1 is a block diagram of a modulated precession nutation control system. The accelerometer output is used to generate time-optimal nutation control thrust pulses of duration π/Ω. However, any precession thrust which has a duration less than π/Ω occuring during the accelerometer negative half-cycle will also reduce nutation, but suboptimally. Only precession pulses that reduce nutation are passed to the thruster.

Two precession modulation nutation control methods are now presented. The first provides continuous precession modulation, and the second allows unmodulated precession until nutation reaches a threshold value at which time the precession is modulated. Figure 2 shows digital simulation results of these control laws applied to a Synchronous Meterological Satellite (SMS) configuration specified as follows: mass = 42.8 slugs, I_x = 244.4, I_y = 246.9, I_z = 97.3, all in slug-ft^2, σ = 0.6039, ω_s = 90 rpm, Ω = 5.69 rad/sec, thrust = 5 lb, moment arm = 3 ft, duty cycle for precession = 1/2 sec (45° of spin period), duty cycle for nutation = 0.552 sec (1/2 nutation period); τ = 180 sec.

Figure 2a shows the behavior for the continuous precession modulation nutation control. Because the gate width, in Eq. (9), varies directly with nutation, the precession increase and nutation decrease will be rapid initially. As the nutation reduces, fewer precession pulses are passed, and the increase in precession reduces to the constant rate shown. Consequently, 2000 sec are required to precess 30°, but after the first 10 sec nutation remains below 2 mrad.

Figure 2b shows the behavior for the nutation control law which allows unmodulated precession until nutation increases to a threshold of 5.48 mrad at which time the precession is modulated. The rate of the precession increase diminishes as the result of precession modulation, and the nutation is held near the threshold level.

In the second case the control law passes all precession pulses until nutation xuilds up to a threshold of 5.48 mrad. Consequently, precession is more rapid and 2100 sec are required to precess the entire 180°.

In contrast, a simulation of a conventional control law allowing nutation control or precession, but not concurrently, for the SMS configuration described requires 1990 sec to precess 180°. The budget is 307.9 sec of fuel. The budget for the modulated precession control law shown in Fig. 2b is 228.9 sec of thruster fuel.

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